

Fatigue assessment strategy using Bayesian techniques

The fulfilment of the probabilistic fatigue characterization of materials based on the S-N field requires the definition of the statistical distribution of the percentile curves of the failure probability, interpreted as a simplification, by their confidence bands. Only in this way a safe design can be ensured according with the structural integrity concept. The Bayesian methodology emerges as a suitable methodology to achieve satisfactorily this aim. Although the Bayesian technique may turn a determinist fatigue model into a probabilistic model, this does not imply to convert an invalid deterministic model in a valid probabilistic model. In the present case, we enhance an already valid probabilistic S-N model into a model to culminate its potential suitability.

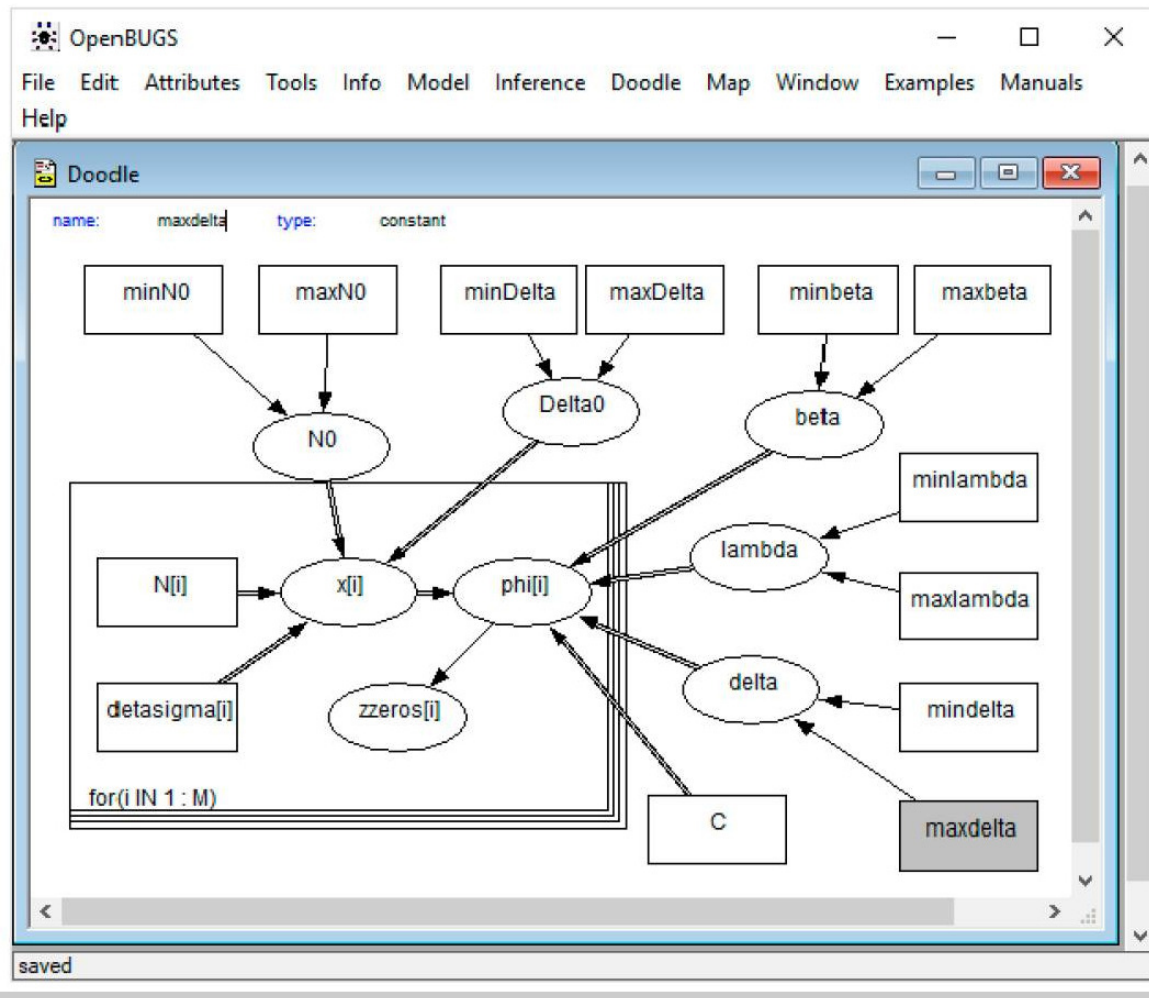


Fig. 1. Block diagram of the Castillo and Canteli model introduced with Graphic Doodle in OpenBUGS.

A short description of the Bayesian methodology

Bayesian methods assume parameters as random and initially a prior distribution of them, reflecting their previous engineer's knowledge on uncertainty. A parametric probability model $X \sim p(x|\theta)$, with parameter θ is defined to represent a variable X and the random parameters θ are associated with a *prior distribution function* $\theta \sim p(\theta|\alpha)$, where α is a vector of the parameter distributions. Based on the model and the distribution functions of the prior parameters, it is possible to make predictions $\tilde{x}$:

$$p(\tilde{x}|\theta) = \int_{\theta} p(\tilde{X}|\theta) p(\theta|\alpha) d\theta \quad (1)$$

The knowledge about the parameters is complemented by random samples, which lead to a *posterior distribution* of the parameters including the two sources of knowledge. Once a sample of size n is obtained from the population being modelled

$$X = (x_1, x_2, \dots, x_n) \quad (2)$$

the predictions are improved by applying the maximum likelihood method by combining the prior distribution and the information from the sample:

$$p(X|\theta) \equiv L(X|\theta) \quad (3)$$

Then, the *posterior distribution* is derived, using the Bayes theorem, and expressed as:

$$p(X|\theta, \alpha) \equiv \frac{p(X|\theta) p(\theta|\alpha)}{p(X|\alpha)} \quad (4)$$

Finally, the *posterior predictive distribution* is obtained, from both, the prior information and the experimental evidence, in the posterior distribution, allowing to make predictions $\tilde{x}$.

$$p(\tilde{x}|X, \alpha) = \int_{\theta} p(\tilde{X}|\theta) p(\theta|X, \alpha) d\theta \quad (5)$$

In Bayesian methods, which are used to both deterministic and probabilistic approaches, an initial family of parametric models is assumed with random, that is, mixtures of those models are used.

The practical application

As a practical example, the extensive fatigue data from the Maennig's classical campaign on Steel C35 steel (similar to SAE 1035 and BS 970) are first evaluated with the Weibull regression model proposed by Castillo and Canteli in which the probability of failure is given as:

$$p_{failure} = 1 - \exp \left\{ - \left[\frac{\log\left(\frac{N}{N_0}\right) \log\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) - \lambda}{\delta} \right]^\beta \right\} \quad (6)$$

where N_0 is the limit number of cycles, $\Delta\sigma_0$ the fatigue limit and λ , δ and β the Weibull location, scale and shape parameters, respectively. The Bayesian methodology permits obtaining the statistical distributions of the model parameters, λ , δ , β , N_0 and $\Delta\sigma_0$, and the stress and lifetime variables, which allows calculating the percentile curves and interpret then as some confidence bands rather than their point estimations.

The implementation is advantageously performed with the OpenBUGS open source software. Figure 1 shows the acyclic graph of the Bayesian network of the

model. Once the code is executed, the program provides the model parameters of the posterior distributions.

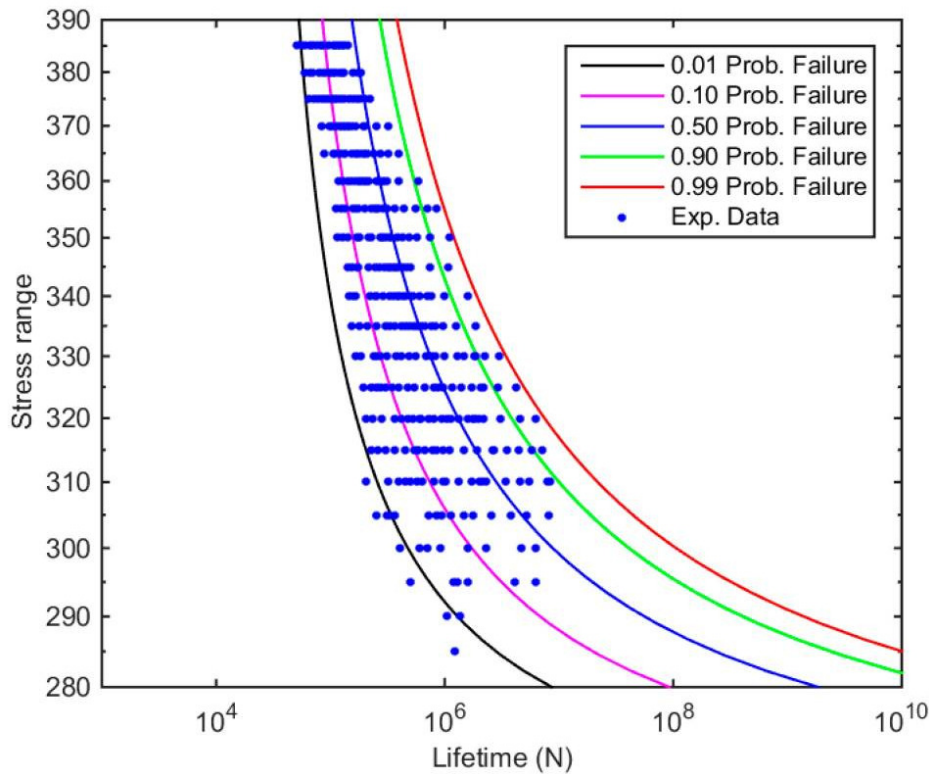


Fig. 2. The 0.01, 0.10, 0.50, 0.90, and 0.99 quantile S-N curves and the 0.01–0.99 confidence intervals for the Maennig data

Finally, the posterior predictive distribution of the model has been used to obtain the 0.01, 0.10, 0.50, 0.90 and 0.99 quantiles of the S-N curves (see lines in Figure 2), and the corresponding 0.01–0.99 confidence intervals (see shaded regions in Figure 2).

As can be seen, the application of Bayesian techniques to the probabilistic regression Weibull model proposed by Castillo and Canteli allows obtaining the probability distribution for any percentile failure curve of the original model, which can be interpreted as confidence intervals.

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