

Mathematicians solve 60-year-old problem

Fermi-Pasta-Ulam chain is a theoretical model of a one dimensional crystal. It consists of point masses connected by nonlinear strings. This model was introduced by Enrico Fermi, John Pasta, Stanislaw Ulam, and Mary Tsingou in 1953, and was investigated with one of the world's first digital computers, and now considered the foundation of experimental mathematics. Fermi, Pasta, Ulam and Tsingou found that the behavior of the system was quite different from what intuition would have led them to expect. Fermi thought that after many iterations, the system would exhibit thermalization, an ergodic behavior in which the influence of the initial modes of vibration fade and the system becomes more or less random with all modes excited more or less equally. Instead, the system exhibited a very complicated quasi-periodic behavior.

The FPU experiment was important both in showing the complexity of nonlinear system behavior and the value of computer simulation in analyzing systems.

Miguel Onorato, Lara Vozella, Davide Proment and Yuri Lvov explained in their paper, entitled "Route to thermalization in the β -Fermi–Pasta–Ulam system" and published in NAS-Proceedings of the National Academy of Sciences of the United States of America explained the reason why researches in the fifties did not observe the thermalization. The reason is very simple - it takes extremely long time for an FPU system to reach thermal equilibrium. Computers in the fifties were just not powerful enough to complete this numerical calculation in reasonable time.

Onorato, Vozella, Proment and Lvov explain why it takes so long to reach thermal equilibrium. According to their arguments, the key lies in a gradual transfer of energy during coincidences of six modes in the system. When precisely six modes interact, the energy is transferred in a nonreversible way. Over many iterations, enough six-wave interactions occur, and enough energy is transferred, to reach complete thermal equilibrium. This conclusion is supported by extensive numerical simulations.

The key approach of our research is to consider the FPU system as a collection of resonantly interacting waves, in other words – waves interact in groups. They have shown that interactions of triads, quartets, and quintets are reversible; in other words, they do not bring the FPU system closer to thermal equilibrium. However, the interaction of waves in sextets does lead to irreversible transfer of energy. It takes the cooperation of six different waves to produce an interaction that is irreversible and, because of that, the process is extremely weak and very slow. That is why it takes so long to approach thermal equilibrium for the FPU system.

Publication

[Route to thermalization in the \$\beta\$ -Fermi-Pasta-Ulam system.](#)

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