Neurons the decision makers

This work is concerned with understanding the synthesis of electric signals in the neural system based on making pairwise comparisons. Fundamentally, every person and every animal is born with the biological talent to compare stimuli from homogeneous things that share properties in space or over time. When not homogenous, they are grouped into homogeneous cluster with a common element from one cluster to the next. Comparisons always need experience to distinguish among things. Pairwise comparisons are numerically reciprocal. If a value is assigned to the larger of two elements that have a given property when compared with the smaller one, then the smaller has the reciprocal of that value when compared with the larger. Because making comparisons requires the reciprocal property, we need mathematics that can cope with division. There are four division algebras that would allow us to use our reciprocals arising from comparisons: The real numbers, the complex numbers, the non-commutative quaternions and the additionally non-associative octonions. Quaternions are an alternative method of handling rotation, besides rotation matrices. The advantage of quaternions over other representations is that they allow interpolation between two rotations. With quaternions we lose the commutative law, and with octonions we also lose the associative law. It was the Irish mathematician W.R. Hamilton who first noticed that multiplication of octonions is not associative. The main thing we are left with is the ability to divide in order to give meaning to proportionality.

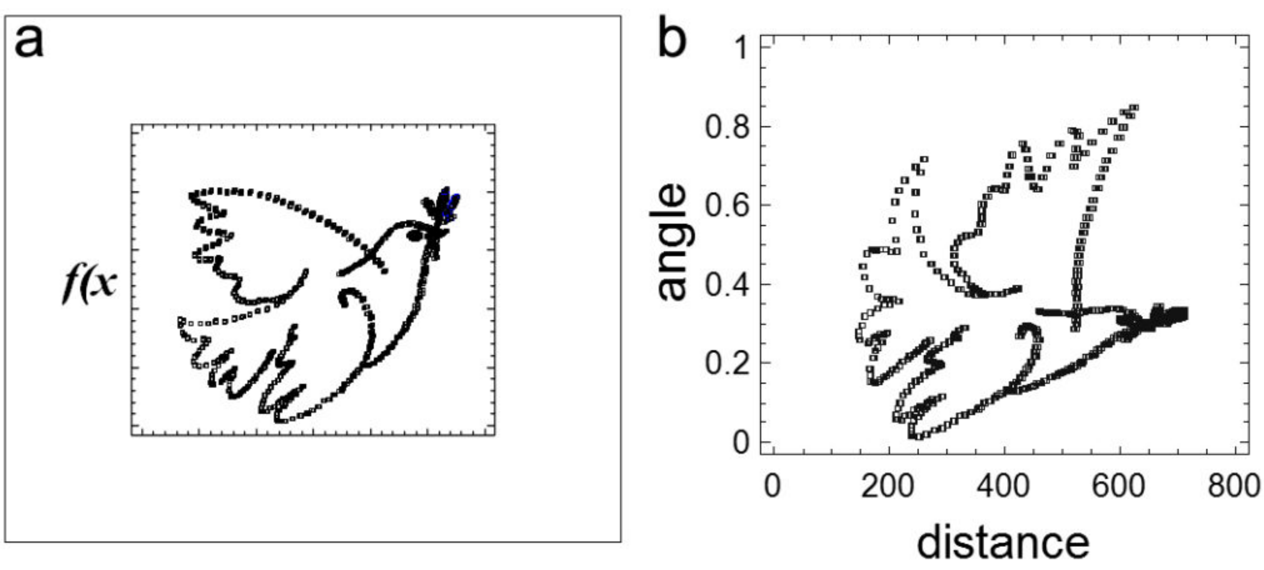

Fig. 1. a. Original of a Picasso drawn bird; the x axis is measured in radians. b. Reproduction using neural firing functions

Rather than inferring function as from electric flows in a network, in this paper we infer the flow from the function. Neurons fire in response to stimuli and their firings vary relative to the intensities of the stimuli. We believe neurons use some kind of pairwise comparison mechanism to determine when to fire based on the stimuli they receive. The ideas we developed about flows are then used to deduce how a system based on the firing works and can be described. Further, the firing of neurons requires continuous comparisons.

To develop a quantitative formula for describing the output of neural pairwise comparisons requires solving Fredholm's equation of the second kind that is a generalization of discrete comparisons to derive the principal eigenvector. Fredholm's equation is satisfied if and only if a simple functional equation has solutions. Our generalization to the continuous case, to Fredholm's equation of the second kind is carried out in two ways. The first is by starting from the beginning to develop the equation. The second is by generalizing the discrete to the continuous formulation. Both yield the same answer. *It turns out that the fundamental proportionality functional equation is a necessary condition for the existence of a solution to Fredholm's equation.* We note that the solution of this functional equation depends on the two parameters and that the ratio involves division and that the proportionality is known to be meaningful in the real and complex domains. Whatever aspect of the real world we consider, sight, sound, smell, heat and cold, at each instant, their corresponding impact our senses numerous times.

The Fourier transform of the real solution of this equation leads to inverse square laws like those in physics. The Fourier transform applied to a complex valued solution leads to Dirac type of firings. Such firings are dense in the very general fields of functions known as Sobolev spaces and thus can be used to represent the very diverse phenomena in and around us. The non-commutative solution in quaternions can be interpreted as rotations in space. The also non-commutative and non-associative solution in octonions has yet to be adequately interpreted outside physics.

A nice examples using the solution of the above mentioned function is shown in Figures 1a and 1b. In the first one we created a 2-dimensional network of neurons consisting of layers. For illustrative purposes, we assume that there is one layer of neurons corresponding to each of the stimulus values. We now make the neurons of the network fire in response to a stimulus such as an image or a sound. To do so, we sampled pictures and sounds to create a set of numerical values that would then be approximated by the firing of neurons. In the case of pictures, we sampled the contour of a picture.

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